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GENERAL SOLUTION OF THE HEAT EQUATION, WAVE EQUATION BY SEPARATION OF VARIABLES

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Abstract :
In this paper, discuss general solution of the heat equation can be found by the method of separation of variables. Some examples in the heat equation article. They are examples of the Fourier series for periodic function f and Fourier transforms for non-periodic function f.
Keywords: Heat equation, Cauchy – kowalevski theorem, partial differential equation, picard theorem.

Introduction :

In mathematics, in the field of differential equations a boundary value problem is a differential equation together with a set of additional restrains called the boundary conditions. A solution to a boundary value problem is a solution to the differential equation which also satisfies the boundary conditions.

The boundary value problems in which the conditions are specified at more than one point. The crucial distinction between initial values problems and boundary value problems is that in the former case we are able to start an acceptable solution at its beginning and just march it along by numerical interaction to its end, while in the present case, the boundary condition at the starting point do not determine a unique solution to start with and a random choice among the solution that satisfy these starting boundary conditions is almost certain not to satisfy the boundary condition at the other specified points.

In mathematics, the Neumann boundary condition is a type of boundary condition, named after Carl Neumann. When imposed on an ordinary or a partial differential equation, it specifies the values that the derivative of a solution is to take on the boundary of the domain.

For an ordinary differential equation, for instance

$Y'' + y = 0$

the Neumann boundary conditions on the interval [a,b] take the form

$y'(a) = \alpha$ and $y'(b) = \beta$ where α and β are given numbers,

In mathematics the Robin boundary condition is a type of boundary condition, named after Vickor Gustave Robin (1855 – 1897). When imposed on an ordinary or a partial differential equation, it is a specification of a linear combination of the values of a function and the values of its derivative on the boundary of the domain, Robin should be pronounced as a French name, although many English – speaking mathematicians anglicize the word.

Robin boundary conditions are a weighted combination of Dirichlet boundary conditions and Neumann boundary conditions. This contrasts to mixed boundary conditions, which are boundary conditions of different types specified on different subsets of the boundary, Robin boundary conditions are also called impedance boundary conditions, from their application in electro magnetic problems.

Robin boundary conditions are commonly used in solving Sturm-Liouville problems which appear in may context in science and engineering used by Agmon (1).

The Born-von Karman boundary condition is important in solid state physics for analyzing many features of crystals, such as diffraction and the band gap. Modeling the potential of a crystal as a periodic function with the Born-von Karman boundary condition and plugging in Schrodinger's equation results in a proof of Bloch's theorem, which is particularly important in understanding the band structure of crustals used by Griffiths (3).

In mathematics a partial differential equation (P.D.E.) is a differential equation that contains unknown multivariable functions and their partial derivative. PDEs are used to formulae problems involving functions of several variables and are either solved by hand, or used to create a relevant computer model.

Partial differential equation (P.D.E.s) are equations that involve rates of change with respect to continuous variable. The configuration of a rigid body is specified by six numbers. Classic domains where PDEs are used include acoustics, fluid flow, electodynamics, and heat transfer.
A partial differential equation (PDE) for the function $u(x_1, x_2 \text{------} x_n)$ is an equation of the form

$$F\left(x_1, x_2 \text{-----} x_n, u, , \frac{\partial u}{\partial x_1} \text{----}, \frac{\partial u}{\partial x_n}, \frac{\partial^2 u}{\partial x_1 \partial x_1} \frac{\partial^2 u}{\partial x_1 \partial x_n}\right) = 0$$

If F is a linear function of u and its derivatives, then the PDE is called linear common example of linear PDE include the heat equation, the wave equation, Laplace's equation, Helmnoltg equation, Klein – Gordon equation, and Poisson's equation from Folland (2).
A relatively simple PDE is

$$\frac{\partial u}{\partial x}(x,y) = 0$$

Hence the general solution of this equation is

$$u(x,y) = f(y)$$

Where f is an arbitrary function of y.

The analogous ordinary differential equation is

$$\frac{\partial u}{\partial x}(x) = 0$$

Which has the solution

u(x)=c where c is any constant

Observation And Discussion :

A solution of a PDE is generally not unique, additional conditions must generally be specified on the boundary of the region where the solution is defined for instance, in the simple example above, the function f(y) can be determined if u is specified on the line x = 0.

EXISTENCE AND UNIQUENESS :

Although the issue of existence and uniqueness of solutions of ordinary differential equation has a very satisfactory answer with the Picard-Lindelof theorem, that is far from the case for partial differential equations. The Caucny-Kowalevski theorem states that the Cauchy problem for any partial differential equation, whose coefficient is analytic in the unknown function and its derivatives, has a locally unique analytic solution. Although this result might appear to settle the existence and uniqueness of solutions, there are examples of linear partial differential equations whose coefficient have derivatives of all orders but which have no solutions at all see Lewy (1957). Even if the solution of a partial differential equation exists and is unique, it may nevertheless have undesirable properties. The mathematical study of these questions is usually in the more powerful context of weak solutions.
An example of pathological behavior is the sequence of Cauchy problems for the laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

With boundary conditions
u(x,0)=0 $\frac{\partial u}{\partial y}(x,0) = \frac{\sin(nx)}{n}$

Where n is an integer
The derivative of u with respect to y approaches o uniformly in x as n increases, but the solution is

$$u(x,y) = \frac{\sin(ny)\sin(nx)}{n^2}$$

This solution approaches infinity if nx is not an integer multiple of π for any nonzero value of y. The

Cauchy problem for the Laplace equation is called dill-posed or not well posed, since the solution does not depend continuously upon the data of the problem. Such ill-posed problems are not usually satisfactory for physical applications, by Taylor (5).
In PDEs, it is common to denote partial derivative using subscripts, That is :

$$u_x = \frac{\partial u}{\partial x}$$

$$u_{xy} = \frac{\partial^2 u}{\partial y\, \partial x} = \frac{\partial}{\partial y}(\frac{\partial u}{\partial x})$$

The equation for conduction of heat in two dimensions for a homogeneous body has

$U_{tt} = c^2 u_{xx}$, it's solution by method of separation of variable of the form u(t,x)=T(t) . X(x)
Then $T'' + k^2c^2T = 0$

$X'' + k^2X = 0$
Where u (x,t) is temperature, and is a positive constant that describes the rate of diffusion. The Cauchy problem for this equation consist in specifying

u (o, x) = f(x),
Where f(x) is an arbitrary function.
General solution of the heat equation can be found by the method of separation of variables, by John (4).

Conclusion :

The general solution of partial differential equation like heat equation, wave equation Laplace equation can be solved by separation of variable and existence and uniqueness of solution.

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References

1) Agmon, Shmuel :Lectures on elliptic boundary value problems, American Mathematical Society, ISBN-0-8218-4910-7 (2010)

2) Folland, Gerald B. : Introduction to partial differential equation (2nd ed.), Princeton university press, ISBN 0-691-04361-2 (1995)

3) Griffiths, Phillips, Harris, : Principles of Algebraic geometry, wiley Joseph interscience. (1994)

4) John, Fritz: P.D.E, Applied Mathematical Science, 1(4th ed.), Springer – verlag, ISSN 0-387-90609-6 (1982)

5) Taylor, Michael E.: Partial D. Equation I Basic theory applied mathematical sciences, 115(2nd ed.) seringer, ISBN 978-1-4419-70 (2011)